COST-EFFECTIVE AND INFORMATION-EFFICIENT ROBUST DESIGN FOR OPTIMIZING PROCESSES AND ACCOMPLISHING SIX SIGMA OBJECTIVES

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KEY WORDS: design of experiments (DOE), propagation of error (POE), robust design

SUMMARY

Standard factorial designs (one array) offer a cost-effective and information-efficient robust design alternative to parameter designs (two-array) made popular by Taguchi. This paper compares these two methods (one-array versus two-array) in depth via an industrial case study. It then discusses advanced tools for robust design that involve application of response surface methods (RSM) and measurement of propagation of error (POE).

PARAMETER DESIGN ADVOCATED BY TAGUCHI (TWO-ARRAY)

In the early 1980's, the methods of Genichi Taguchi became popular as a tool for quality improvement, especially in the automotive industry. Taguchi provided a focus on robust design that reached fruition in the Six Sigma movement. The basic philosophy of robust design is that it's desirable to develop processes that consistently produce product to target with minimal variation.

To achieve this objective, Taguchi advised that input variables be divided into two categories:

1. Control factors, for example the adjustments a technician makes to your office copier.
2. Noise factors, for example the humidity that fluctuates in your office environment.

Ideally, processes will be adjusted via the control factors to be insensitive to the noise factors, whose variation presumably cannot be eliminated. For example, the office copy machine should handle paper properly during humid summer months as well as in the winter, when conditions are dry and prone to static electricity. To accomplish these objectives, Taguchi advocated planned experimentation using a layout called "parameter design" (Ross, 1988, page 170).

To illustrate parameter design, let's look at an industrial example - development of a new method for attaching an elastomeric connector to a nylon tube. We won't show any numerical data or analysis because, as will be shown later, there's a more efficient approach to accomplishing the same objectives for robust design. Data and analysis will be included at this stage of the paper.

In this case, the objective is to consistently deliver a specified pull-off force via three control factors:

A. Sanding (affects the smoothness/friction of the interfering surfaces – inner connector to outer tube)
B. Connector wall thickness
C. Insertion depth.

The following noise variables, all related to storage of the connector, are not normally controlled during production, but they will be included in the experiment:

D. Time
E. Temperature
F. Relative humidity
G. Ultraviolet (UV) exposure (depends on opacity of packaging as well as exposure to various forms of light).

Factor levels should be selected from the extremes, in order to maximize the possible effect seen. The practicality of this was recognized by W. Edwards Deming, who said "start with strata near the extremes of the spectrum of possible disparity…as judged by the expert in subject matter." In this case, the subject matter experts reviewed the historical records on storage of the tubing and identified the extreme environment conditions before establishing the experimental ranges.

In general, a parameter design is composed of two parts:

1. An inner array of control factors
2. An outer array of noise variables.

Table 1 shows the parameter design that we propose for the connector case. As noted below, it varies somewhat from that shown by Taguchi and his disciples (such as Ross).

1. The control factors are studied via a full two-level factorial design (2^3), which provides complete resolution of all effects. The 8 runs are listed in standard ("Std.") order, which differs from that used by Taguchi.
2. The noise variables go into a standard half-fraction two-level design (2^{4-1}). This design resolves main effects fairly well, but it leaves two-factor interactions aliased (Anderson and Whitcomb, 2000, page 95). The 8 runs in this portion of the parameter design (the outer array) are also listed in standard order.

We altered the Taguchi form of parameter design to improve it and achieve consistency with the one-array design alternative discussed later in this paper.

The levels in both arrays are coded minus (–) and plus (+) for the low and high levels; respectively, rather than the numbers 1 and 2 used by Taguchi for what he calls the first and second levels. The number of runs in the parameter design for the connector case totals 64 (8 inner x 8 outer).

			Outer Array (noise variables)										
			Std →	1	2	3	4	5	6	7	8		
			D	–	+	–	+	–	+	–	+		
			E	–	–	+	+	–	–	+	+		
			F	–	–	–	–	+	+	+	+		
			G	–	+	+	–	+	–	–	+		
Inner Array (control factors)				**Raw Data**								**Responses**	
Std	A	B	C									Mean	Std. Dev.
1	–	–	–										
2	+	–	–										
3	–	+	–										
4	+	+	–										
5	–	–	+										
6	+	–	+										
7	–	+	+										
8	+	+	+										

Table 1: Parameter design for connector study

We advise that whenever practicable the actual runs in any DOE be done in random order, which avoids possible confounding of effects with time-related lurking variables, such as machine wear and the like. In any case, after collecting all the raw data, the experimenter should then calculate the mean and standard deviation ("Std. Dev."). We recommend that these summary statistics be analyzed separately before putting them in the form of a ratio, such as Taguchi's signal-to-noise, thus preserving valuable information that might be lost otherwise. The significant factors, identified via analysis of variance (ANOVA), will fall into one of four classes (Ross, page 175) affecting:

I. Both average (mean) and variation (standard deviation)
II. Variation only
III. Average only
IV. Nothing.

The strategy is to pick proper levels of class I and II factors to reduce variation. Then adjust the class III factor(s) to bring the average to the target level.

Parameter design neatly accomplishes the goal of determining a setup of control factors that will be robust to variations caused by noise factors. However, as demonstrated below, this experiment can be designed in such a way that reveals essentially as much information on the effects of control factors and noise variables, but with only half the data (and work needed to generate it!).

MORE EFFICIENT ALTERNATIVE DESIGN THAT COMBINES ALL FACTORS INTO ONE ARRAY

Let's look at an alternative design option for the connector study. It combines all the factors, control and noise, into one two-level factorial array (see Table 2). This design, a standard quarter-fraction (2^{7-2}), requires only 32 runs.

Std	A	B	C	D	E	F	G	Pull-off Force
1	–	–	–	–	–	+	+	15.61
2	+	–	–	–	–	–	–	14.56
3	–	+	–	–	–	–	–	14.52
4	+	+	–	–	–	+	+	16.62
5	–	–	+	–	–	–	–	20.96
6	+	–	+	–	–	+	+	26.29
7	–	+	+	–	–	+	+	17.01
8	+	+	+	–	–	–	–	15.43
9	–	–	–	+	–	–	+	23.09
10	+	–	–	+	–	+	–	27.64
11	–	+	–	+	–	+	–	18.85
12	+	+	–	+	–	–	+	16.60
13	–	–	+	+	–	+	–	27.69
14	+	–	+	+	–	–	+	26.69
15	–	+	+	+	–	–	+	22.52
16	+	+	+	+	–	+	–	26.66
17	–	–	–	–	+	+	–	13.82
18	+	–	–	–	+	–	+	10.39
19	–	+	–	–	+	–	+	14.38
20	+	+	–	–	+	+	–	19.41
21	–	–	+	–	+	–	+	19.57
22	+	–	+	–	+	+	–	24.39
23	–	+	+	–	+	+	–	19.99
24	+	+	+	–	+	–	+	17.57
25	–	–	–	+	+	–	–	14.75
26	+	–	–	+	+	+	+	18.75
27	–	+	–	+	+	+	+	15.66
28	+	+	–	+	+	–	–	14.12
29	–	–	+	+	+	+	+	20.02
30	+	–	+	+	+	–	–	17.68
31	–	+	+	+	+	–	–	21.21
32	+	+	+	+	+	+	+	26.97

Table 2: Alternative (single array) design for connector study

The complete alias structure for this design can be seen in textbooks (Anderson and Whitcomb, 2000, Appendix 2-4) so we won't show it all here. All main effects can be estimated clearly because they're aliased only with interactions of three or more factors, which by common practice are assumed to be negligible. Furthermore, all two-factor interactions involving A, B or C (the control factors) are also aliased only with three-factor interactions, so they can be relied upon as well. The only aliasing of any concern occurs among the noise variables - D, E, F and G:

- [DE] = DE + FG
- [DF] = DF + EG
- [DG] = DG + EF

Normally, any aliasing of two-factor interactions might be considered somewhat risky, especially in this case, considering the nature of the noise variables: time (D), temperature (E), humidity (F) and ultraviolet exposure (G). All of these factors might interact in chemical reactions that degrade the elastomeric connector material. It would be nice to clearly identify all possible interactions, but given that the noise variables will not be controlled in the production environment, why waste time studying this?

Let's see what the statistical analysis reveals and whether the objectives of robust design can be accomplished. (Disclaimer: The response data ("Pull-off Force") comes from a simulation designed to generate a thought-provoking analysis, so any resemblance to a real manufacturing process is purely coincidental.) We begin our story by viewing a half-normal plot of the effects (Figure 1), produced with the aid of DOE software (Helseth, et al, 2001). It shows the relative impact of the 31 effects that can be estimated from the 32 unique runs in the design.

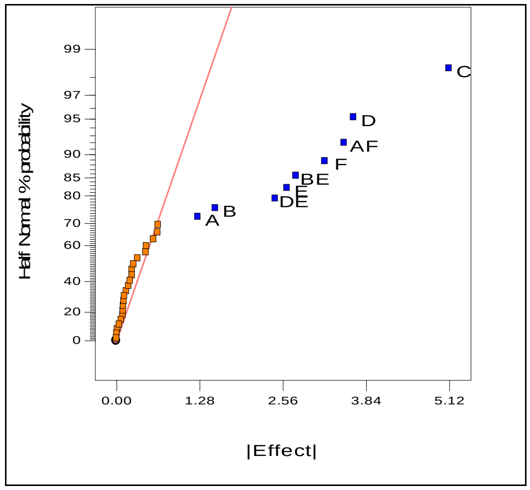
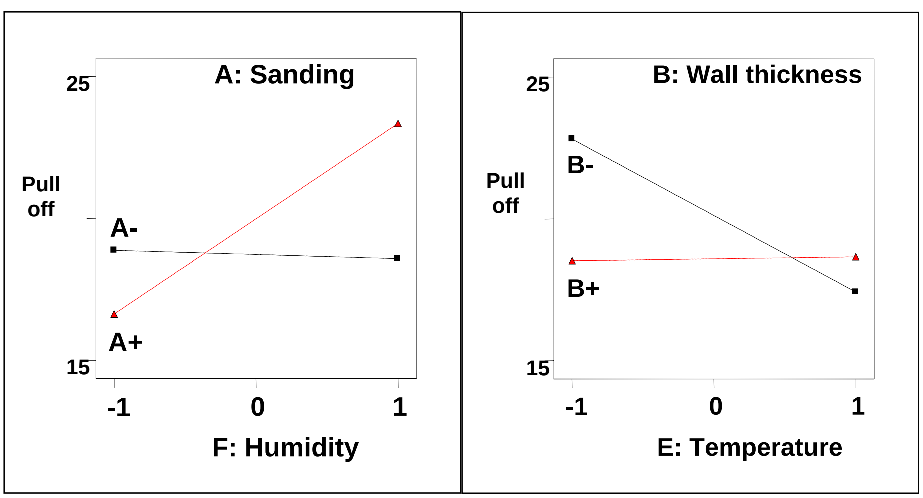


Figure 1: Effects plot

The labeled effects are statistically significant with more than 99.99% confidence according to analysis of variance (ANOVA). (We did a good job on the simulation!) Before interpreting the results, it will help to categorize factors as follows:

1. Control factors involved in interactions with noise variables (A with F, B with E)
2. Control factors not involved in interactions with noise variables (C)
3. Noise variables not involved in interactions with control factors (D, E)
4. Control factors that have no effect (none in this case)
5. Noise variables that have no effect (G)

First of all, let's look at the interaction graphs for AF and BE (Figures 2 and 3).



Figures 2a, 2b: Interactions of control by noise – AF (left) and BE (right)

When displaying interactions in this category (control by noise), interpretation comes easier by putting the noise factor on the X-axis. To find the conditions that will be most robust, simply find the flats – the operating lines that remain least affected by noise. In this case, the flats are found with sanding set at the low level (A–) and wall thickness at its high level (B+), thus making the process robust to variations in humidity (F) and temperature (E); respectively.

Setting A and B to minimize variation make these two factors inflexible for controlling the response. That's where factor C (insertion depth) comes into play. It falls into the second category of factors that do not interact with noise variables. The effect of C is shown in Figure 3.

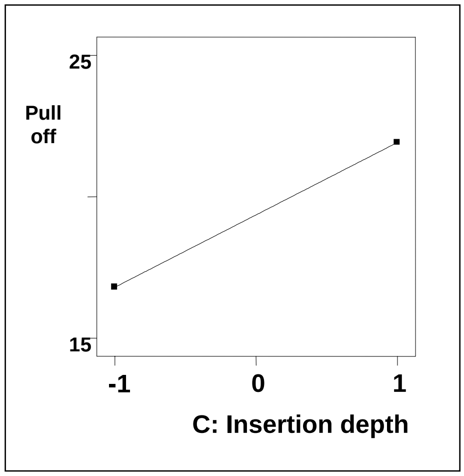


Figure 3: Main effect of factor C, which does not interact with any noise variables, thus making it good for control

Using this figure as an operating curve, the experimenter can adjust factor C upwards or downwards to increase or decrease the response (pull-off force) so it meets the required specification (not shown, but presumably a range – not too low or too high).

The last thing to look at are noise variables that do not interact with control factors. In this case, variables D (time) and E (temperature) fall in this category as well as their interaction effect (DE). Remember that DE is aliased with FG in this design, so we can't be sure that DE really does create an effect. However, one of the parent terms of FG, G (UV exposure), does not exhibit significant effects. On the other hand, both D and E are significant, so it's a good bet that their 'child,' DE, really is the significant interaction rather than its alias - FG.

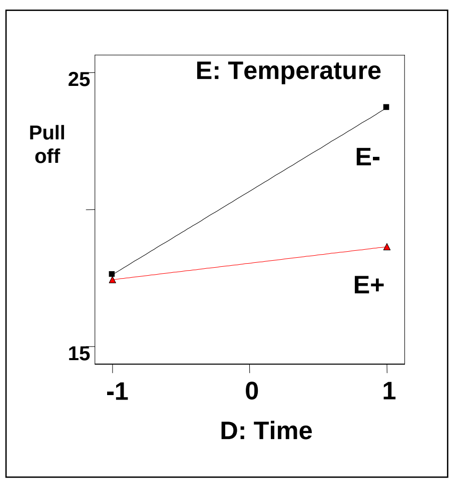


Figure 4: Interaction of noise by noise variables - DE

What, if anything, can be done when a process is significantly affected by noise variables? The obvious answer is to impose control. For example, in this case nothing can be done about the time (D) the connector stays in storage (variable D), but temperature (E) could be controlled with the proper equipment. The high level (E+) moves the response to the flat line on the interaction graph, which will be robust to the affects of varying time, so it would be wise to invest in some space-heating (it gets cold in Minnesota!). This will put the process that much closer to conformance with Six-Sigma objectives.

This brings our story to a satisfying conclusion. The experimenter now knows how to control the pull-off force of the elastomeric connector from the nylon tube and minimize the impact of noise variables. The two-array parameter design most likely would have generated the same results, but with twice the effort (64 runs versus the 32 runs in the single-array 2^{7-2} design). The information on interactions of noise variables is a big bonus obtained by choosing the 2^{7-2} option for robust design. This detail would typically get overlooked by experimenters following procedures advocated by Taguchi because the focus remains only on the control factors.

As a postscript to the story, note that there may be control factors that are not significant (category 4). You may be tempted to set these factors at levels that are most economical or convenient for operation, but that would not be consistent with the objectives of robust design. It's best to leave the insignificant factors at their mid-level in case they might vary. Hopefully such variation will occur within the experimental range and thus should not affect the process.

Similarly, some noise variables, such as the exposure to UV in this case, may not create significant effects on the response (category 5). At the very least, this might justify leaving these variables uncontrolled. For example, in this case, it appears that there would be no advantage to spending money on new, UV-resistant packaging for the connectors.

ADVANCED TOOLS FOR ROBUST DESIGN: RESPONSE SURFACE METHODS (RSM) AND MEASUREMENT OF PROPAGATION OF ERROR (POE)

A more advanced approach to robust design makes use of response surface methods (Myers and Montgomery. 2002). Figure 5 shows an example of a response surface generated from the interaction DE in the connector case.

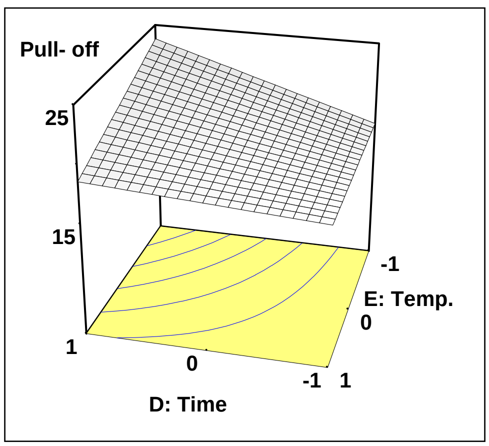


Figure 5: Response surface plot of interaction DE

Consider how variations in time get transmitted to the response (pull-off force). These variations will be substantially reduced at the flatter portion of the surface (in front), where temperature gets set at its high (+1) level. A mathematical tool called propagation of error (POE) can be applied to quantify the error transmitted via the response surface from the variations in input factors, which must be determined beforehand from repeatability studies, SPC or ANOVA on prior DOE data (Anderson and Whitcomb, 1996).

To get a feel for POE, consider a very simple, fun and practical example: driving through rush hour traffic (Anderson, 2002). Figure 6 shows how timing is everything if you want to get to work consistently on time. It shows how drive time (in minutes) varies as a function of the time of departure (minutes after 6:30 A.M.). Notice the waviness that occurs due to traffic patterns. Assume that the driver won't leave exactly at the targeted departure time – this will vary over a 10 minute window. That variation in departure will be transmitted via the response curve to the drive time. At T1 this variation is very slight (see the small arrow on the drive time axis). However, only a short while later at T2, the variation gets amplified (see the big arrow on drive time) by the steep curve that results from daily traffic jams in the heart of the rush hour.

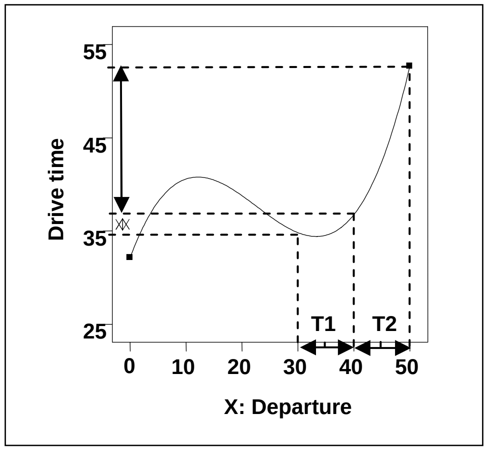


Figure 6: Variation transmitted from input factor (departure time) to output response (drive time)

RSM generates polynomial models to fit data such as that collected on drive time. The model used to create the curve in Figure 6 is cubic:

1. Drive time = $Y = 32.13 + 1.72X - 0.102X^2 + 0.00159X^3$

When the model reveals curvilinear relationships between controllable factors and responses, such as the drive time case, transmitted variation can be reduced by moving to plateaus. The catch-phrase for robust design, as stated earlier, is "find the flats." These can be located by generating the POE, which involves some calculus (Anderson and Whitcomb, 1996). The first step is calculation of the response variance (σ_y^2):

2. $\sigma_y^2 = (\delta Y/\delta X)^2\, \sigma_X^2 + \sigma_e^2$

where σ_X^2 is the variance of the input factor X and σ_e^2 is the residual variance which comes from the analysis of variance (ANOVA). In the drive time case these values are:

3. $\sigma_X^2 = (5 \text{ minutes})^2 = 25$
4. σ_e^2 = Mean square residual from ANOVA = 5.18

The partial derivative of the response Y with respect to the factor X ($\delta Y/\delta X$) for the drive time model is:

5. $\delta Y/\delta X = 1.72 - 0.204X + 0.00477X^2$

The POE is conveniently expressed in the original units of measure (minutes of drive time in this case) by taking the square root of the variance of response Y via this final equation:

6. $\text{POE} = [\sigma_y^2]^{0.5} = [(\delta Y/\delta X)^2\, \sigma_X^2 + \sigma_e^2]^{0.5} = [(1.72 - 0.204X + 0.00477\, X^2)^2 * 25 + 5.18]^{0.5}$

With the aid of software (Helseth, et al, 2002), the POE can be calculated, mapped and visualized. Figure 7 shows the POE for the drive time case study. It exhibits two local minima, one for the peak at 11.5 (a high 'flat' point) and one for the valley at 31.5 (a low 'flat' point).

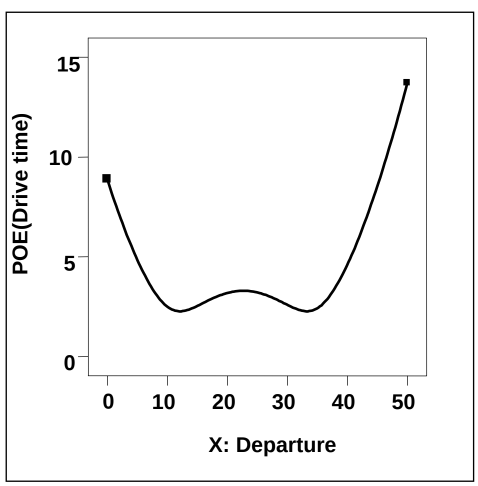


Figure 7: POE of drive time versus departure time

The drive time example involves only one factor. Generally, robust design via RSM involves more than one factor. In fact, for purposes of robust design, the experimenter should try to include control factors and noise variables thought to be significant based on past experience (prior DOE) or subject matter knowledge. Ideally, the analysis will reveal that factors affect the response in various ways:

1. Non-linear (curved surface)
2. Linear

The non-linear factors, such as temperature in the connector case, should be set at levels that minimize POE. On the other hand, as shown in Figure 8, the variation transmitted by a linear factor (X2) will be constant, so POE becomes irrelevant.

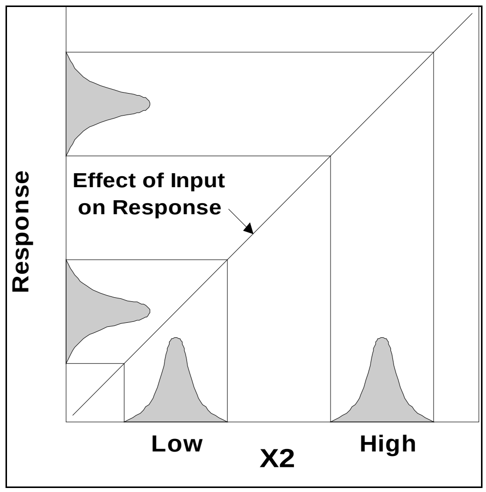


Figure 8: Variation transmitted from a linear factor (X2)

Therefore, linear factors can be freely adjusted to bring the response into specification while maintaining the gains made in reducing variation via POE on the non-linear factors. The ideal case is represented by Figure 9. The top curve represents production before applying the tools of robust design.

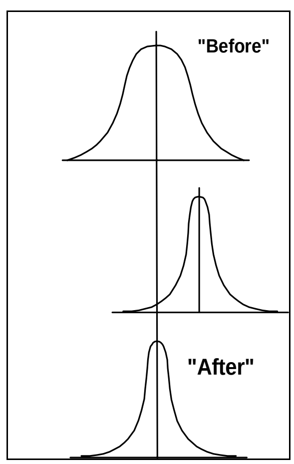


Figure 9: Achieving variability reduction by intelligent manipulation of factor levels

With information gained from POE analysis, the experimenter can adjust the non-linear factor(s) to minimize process variation. This tightens the distribution as shown by the middle curve in Figure 9. However, average response now falls off target. Therefore, a linear factor, such as X2 in Figure 8, must be adjusted to bring the process back into specification (the “After” curve). Thus, the mission of robust design and Six Sigma is accomplished.

CONCLUSIONS

As shown by example, standard two-level fractional factorials (one array, 2^{k-p}) are more efficient and provide more information than the two-array parameter designs advocated by Taguchi. For any given parameter design, it will be worthwhile to investigate alternative 2^{k-p} designs to see if the number of runs can be reduced and/or more effects resolved, particularly those involving interactions of noise variables. If control can be imposed on certain noise variables, the effects of other noise variables might be reduced. This information would likely be lost in the overly simplistic approach of parameter design, which somewhat arbitrarily distinguishes control factors from noise variables.

Experimenters should consider how much variation can be anticipated in all the variables. This data, typically entered in the form of standard deviations, can then be made use of in the calculation of propagation of error (POE). POE, in conjunction with response surface methods (RSM), facilitates the search for robust operating regions – the flats where variations in input factors do not get transmitted much to the response. The end-result of applying these advanced tools will be in-specification products that exhibit minimal variability – the ultimate objective of robust design and Six Sigma.

ACKNOWLEDGEMENTS

The authors thank their colleague Patrick Whitcomb for many valuable suggestions and much of the source material for this paper.

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