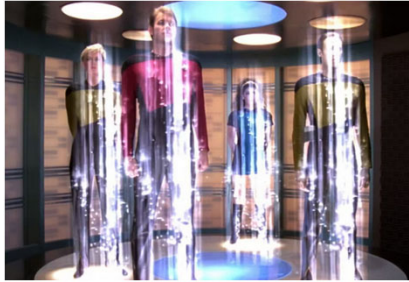




Design of Experiments The Science of *Test*



Wanted: A Star Trek Transporter

presented to:
StatEase DOE Summit
Oct 6, 2022

Greg Hutto, Chief Ops Analyst - 96 Ops Group, Eglin AFB, Florida gregory.hutto@us.af.mil



Short Bio



Mr. Gregory Hutto is Wing Operations Analyst for the 96th Test Wing, responsible for embedding designed experiments as the principal test method for several hundred tests each year. He teaches an extensive series of short courses in test methods to all testers in the Wing from the Wing Commander to our 520 scientists and engineers. As a LtCol in the USAF Reserves, he served as senior military advisor to AF Operational Test & Evaluation Center Test Support Director and as special advisor for test design to the AF Flight Test Center commander at Edwards AFB, California. Mr. Hutto is a distinguished graduate of the US Naval Academy in Operations Research and holds a Master's in the same field from Stanford University. Over the past 43 years, he has served in nearly every branch of test and evaluation from laboratory basic science to joint operational field testing. He would like to publicly repent of his 11 years of testing without the benefits of the principles of well-designed experiments.

He is proud of his two children who make their livings in the sky – one serves martinis; the other mayhem. Ariel is a Delta Flight Attendant and Daniel is a F-15E Strike Eagle WSO - bomb/nav. He is inordinately proud of his highly educated wife, Dr. Deb, with 6 degrees.





Abstract

Wanted: A Star Trek Transporter

Until a Craig's List ad succeeds, our Air Force aircraft must traverse the sky searching for targets. Sensor performance depends on sensor-target geometry so statistical models include range, azimuth, elevation, etc. Without our transporter, aircraft position is a **sequential** hard-to-change (HTC) factor. That is, short ranges follow long ranges inbound towards the target and vice versa as we recede from the target. Result: a physics-imposed design grid with several sequential ranges for each altitude.

In addition to these geometric variables we have several other easy-to-change (ETC) "modal" variables:

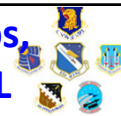
- side of the aircraft (is performance symmetric?),
- electromagnetic spectrum used,
- target tracking algorithm selected,
- target type, etc.

The design challenge is that the HTC values are determined and sequenced. How shall we mate the large number of combinations of the modal variables to the fixed geometric values so that the resulting design has excellent modeling properties - orthogonality, power, low variance inflation, good prediction variance, etc.? A fractional split plot design invariably leaves "holes" where the aircraft occupies positions when no points are to be taken - an obvious inefficiency.

This paper describes the algorithm we derived that takes advantage of several of Design Expert's features including building and importing custom design and using DX's excellent evaluation features to ensure that the modeling properties are acceptable. Because the resulting design incorporates both the whole plot geometric design stitched to an (often) optimal blocked design for the modal variables, the resulting design can be viewed as a "Franken-design", a term of art we invented to describe designs built from individually crafted pieces and merged. The author introduced Franken-designs last year at the StatEase Summit!



Eglin AFB Florida – Weapons, Special Ops, Software, Data Links, Fighters & e-VTOL



The collage includes:

- A map of Florida with Eglin Air Force Base highlighted in green.
- A photograph of a large transport aircraft (C-17 Globemaster III) in flight over the ocean.
- A photograph of a smaller aircraft (eVTOL) in flight.
- A diagram of a flight test and training range with various test cells labeled (e.g., EFTA-01, EFTA-02, EFTA-03, EFTA-04, EFTA-05, EFTA-06, EFTA-07, EFTA-08, EFTA-09, EFTA-10, EFTA-11, EFTA-12, EFTA-13, EFTA-14, EFTA-15, EFTA-16, EFTA-17, EFTA-18, EFTA-19, EFTA-20, EFTA-21, EFTA-22, EFTA-23, EFTA-24, EFTA-25, EFTA-26, EFTA-27, EFTA-28, EFTA-29, EFTA-30).

We're recruiting great experimentalists ... or at least enthusiastic ones!



U.S. AIR FORCE

AF Ops Research Recognized



US Air Force awarded the 2017 Institute for Operations Research and the Management Sciences (INFORMS) Prize for Operations Research



The INFORMS Prize honors effective integration of advanced analytics and operations research/management sciences (OR/MS) into organizational decision making. The award is given to organizations that have repeatedly applied the principles of OR/MS in pioneering, varied, novel, and lasting ways. Past recipients of the award include General Motors, Intel, UPS, HP, IBM, Ford, Procter & Gamble, and GE Research.

U//FOUO

Integrity - Service - Excellence

25



It All Started with Beer!



- Gosset – Dublin Guinness Brewery circa 1899-1908
- Brewing stouter porter, stout, or Guinness
- Guinness decided to make brewing scientific
- Hired Gosset, a chemist, -- self-taught in statistics and worked with Karl Pearson developing comparative test
- Paper on **Probable Error of A Mean** forms the basis for our statistical “Student t” tests
- Temperature, robust hops, and Archer breed of barley -- plus experimentation key in Guinness findings!
- Gosset’s paper corrected by RA Fisher who later extended 2 sample t to M-way/k-Variable ANOVA



W. S. Gosset, 1899

BIOMETRIKA:

THE PROBABLE ERROR OF A MEAN.

By STUDENT

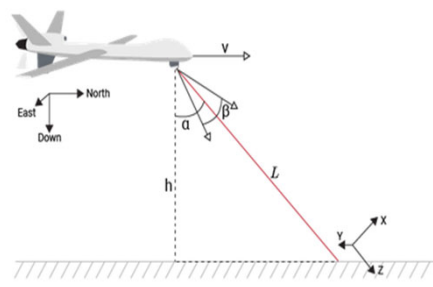




Sensor Cross-Blocks Problem



Critical features of target coordinates problem



1. Estimating target coordinates is a frequent, DOD-wide problem
2. The target battlespace is large, with many variables
3. The response variable is skew with unstable variance
4. We have both hard-to-change factors and restrictions on randomizing execution order (no transporter) ...

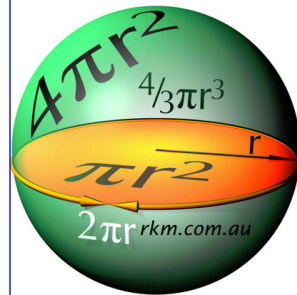
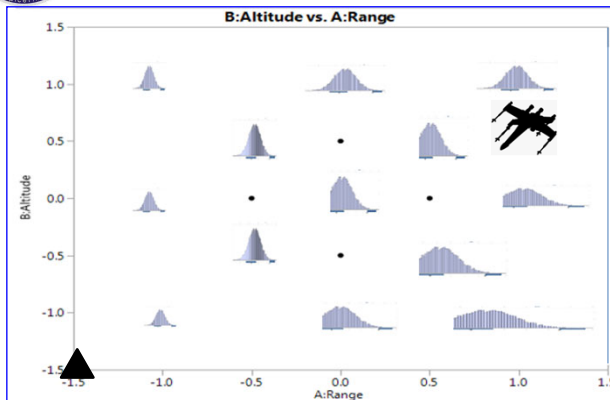
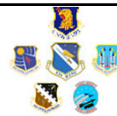
Source (line drawing): <https://www.vectornav.com/> 1 Oct 22



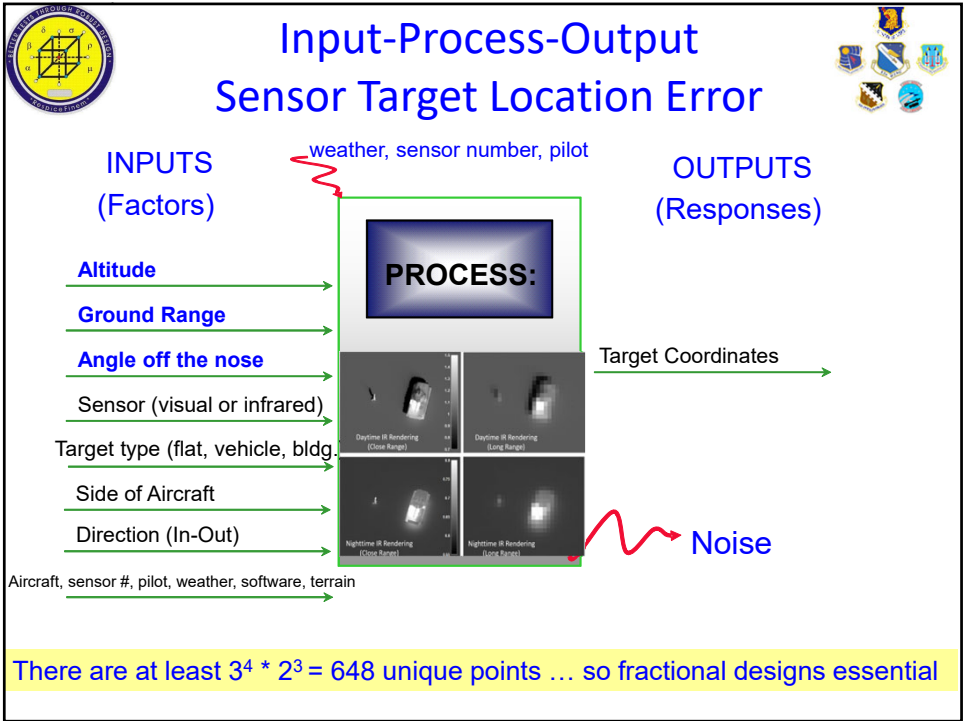
Target Coordinates are a common test problem as we frequently roll out new “operating systems”



Complications – response distribution changes with geometry



- Coordinate error increases as slant range (radius) squared
- At low graze angle, long slant range, errors are more skew
- And, we are challenged to model percentiles (50th, 90th)
- But today let’s focus on the design issues...



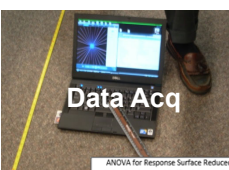
You CAN do this at home... laser safety procedures ... of course...

Step 1a: Experiment Planning and Power Analysis


Sample Factors	Factor Label	Numeric or Categorical	Minimum	Maximum
Height Above Target	A	3"	3"	36" (Waist)
Horizontal Range	B	1'	1'	7.5'
Angle off Nose	C	0	0	90 deg
Sensor*	D	Eyes	Eyes	Ears
Sensor Quality	E	Mono (Mono)	Bath (Stereo)	
Hand	F	Dominant	Off	
Aim Prop	G	Unstable	Stable	
Operator Orientation	H	1 spin	3 spins	
Target offset	J	0	2 feet	
Aim assist	K	No extension (pistol)	Dowel extension (rifle)	

Planning

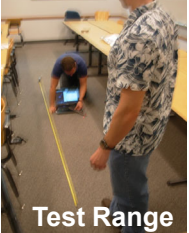
Data Acq



Test Ops



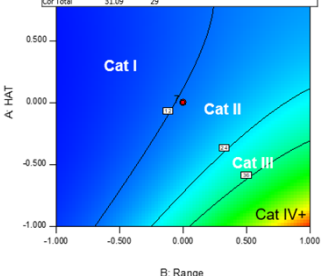
Test Range



Analyze & Interpret

ANOVA for Response Surface Reduced Quadratic Model
Analysis of variance table (Partial sum of squares - Type III)

Source	Sum of Squares	df	Mean Square	F	p-value
Block	0.27	1	0.27		
Model Total	24.74	5	4.95	18.69	< 0.0001 significant
A:Height	6.27	1	6.27	23.68	< 0.0001
B:Range	19.02	1	19.02	71.85	< 0.0001
A*B	0.45	1	0.45	1.65	0.2105
Residual	6.09	23	0.26		
Lack of fit	5.28	18	0.29	1.82	0.2625 not significant
Pure Error	0.80	5	0.16		
Cor Total	31.09	29			



- We have a classroom physical laser-pointer TLE lab you can execute as a science experiment ... complete with .mp3 files



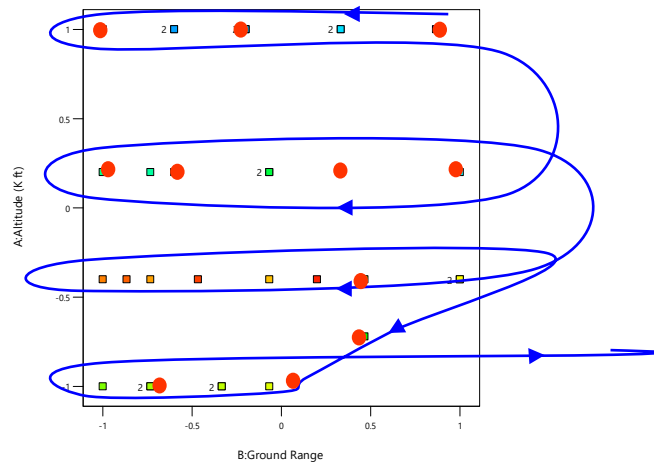
First we'll deal with geometry, then modes



Source: <https://www.quora.com>



Transporter vs. 2022 Tech



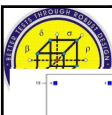
- Gold standard is a randomized and blocked experimental design **but...**
- Until Craig's List ad is answered we must occupy space sequentially



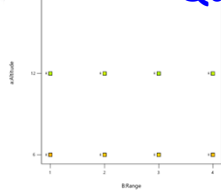
Geometric Design Looks Like This

- Constrained to a sequential vs. random order
- First problem is this imposes a split-plot structure on us
- Ameliorated by
 - a. We understand the physical model
 - b. We can afford several replicates at the Whole Plot level
- Second Problem is what to do with Easy-to-Change (ETC) variables?

	Point	Altitude mF	Ground Range kC	Angle to Target	
	1	10	15	0	
	2	10	12	0	
	3	10	8	0	inbound
	4	10	4	0	inbound
	5	10	4	180	
	6	10	8	180	outbound
	7	10	12	180	outbound
	8	10	15	180	outbound
	9	15	15	0	
	10	15	12	0	inbound
	11	15	8	0	inbound
	12	15	4	0	inbound
	13	15	4	180	
	14	15	8	180	outbound
	15	15	12	180	outbound
	16	15	15	180	outbound
	17	20	15	0	
	18	20	12	0	inbound
	19	20	8	0	inbound
	20	20	4	0	inbound
	21	20	4	180	
	22	20	8	180	outbound
	23	20	12	180	outbound
	24	20	15	180	outbound
	25	10	15	45	
	26	10	12	45	inbound
	27	10	8	45	inbound
	28	10	4	45	inbound
	29	10	4	135	
	30	10	8	135	outbound
	31	10	12	135	outbound
	32	10	15	135	outbound
	33	15	15	45	
	34	15	12	45	inbound
	35	15	8	45	inbound
	36	15	4	45	inbound
	37	15	4	135	inbound
	38	15	8	135	inbound



Split Plot Optimal Designs Jump Quickly to Mind ... but ohhhh...



Design evaluates nicely

Term	Standard Error*	Error df†	VIF	Restricted VIF	Power
Whole-Plot					
a	0.3150	14	1.01065	1.00161	83.9 %
c	0.2588	14	1.02106	1.01395	94.8 %
d	0.2588	14	1.02229	1.01416	94.8 %
Subplot					
B	0.1385	72	1.01149	1.0077	99.9 %
E[1]	0.1485	72			99.9 %
E[2]	0.1489	72			
F	0.1036	72	1.0045	1.00303	99.9 %
aB	0.1699	72	1.02738	1.02198	99.9 %
B ²	0.2340	72	1.00969	1.00469	98.8 %

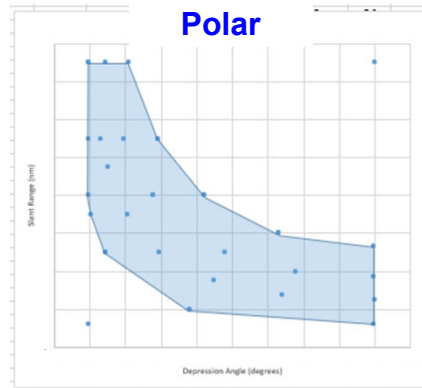
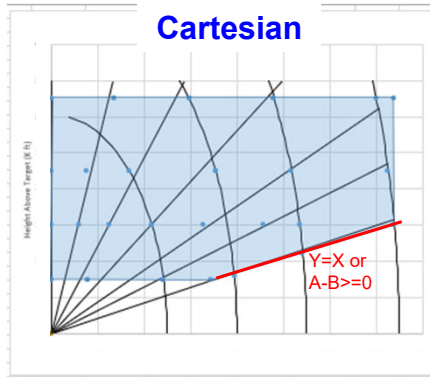
Group	Run	Factor 1 a:Altitude	Factor 2 B:Range	Factor 3 c:Direction	Factor 4 d:Freq	Factor 5 E:Target	Factor 6 F:Side	Response 1 R1
	1	1	18	4	In	Lo	Vespa	Right
	1	2	18	1	In	Lo	Vespa	Right
	1	3	18	1	In	Lo	TinyHouse	Right
	1	4	18	1	In	Lo	TinyHouse	Left
	1	5	18	4	In	Lo	GoatPath	Left
	2	6	6	4	In	Lo	TinyHouse	Left
	2	7	6	2	In	Lo	GoatPath	Left
	2	8	6	1	In	Lo	GoatPath	Right
	2	9	6	3	In	Lo	Vespa	Left
	2	10	6	4	In	Lo	Vespa	Right
	2	11	6	2	In	Lo	TinyHouse	Right
	3	12	18	4	Out	Hi	GoatPath	Right
	3	13	18	1	Out	Hi	GoatPath	Left
	3	14	18	1	Out	Hi	TinyHouse	Right
	3	15	18	3	Out	Hi	Vespa	Left
	3	16	18	2	Out	Hi	TinyHouse	Right
	4	17	12	4	In	Hi	GoatPath	Left
	4	18	12	4	In	Hi	TinyHouse	Right
	4	19	12	1	In	Hi	Vespa	Right
	4	20	12	2	In	Hi	TinyHouse	Left
	4	21	12	1	In	Hi	GoatPath	Right
	5	22	12	4	Out	Hi	GoatPath	Left
	5	23	12	4	Out	Hi	Vespa	Right
	5	24	12	1	Out	Hi	Vespa	Left
	5	25	12	2	Out	Hi	TinyHouse	Right
	5	26	12	3	Out	Hi	GoatPath	Right
	5	27	12	1	Out	Hi	TinyHouse	Right
	6	28	18	3	Out	Hi	GoatPath	Right

Any issues? Lots!

- Substantial by-hand re-ordering to make it somewhat executable...
- Fractional “holes” make us look dumb...
- Hovering fighters anyone? Climb-Dive-Climb again...
- “Ummm ... pilot take next two then skip 1 then skip 3 then go back one...”



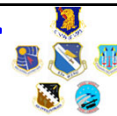
BTW - Which Coordinate System Shall We Use to Build our Statistical Model?



- Represent with Cartesian (with Linear Constraint) or
- Polar Coordinates with physics constraints (no 60kft over target)?
- Objective decision is made easy with DX comparing Design Evaluations



Comparing Cartesian with Polar in Design Expert's Evaluation



Power-VIF

Cartesian

Term	Standard Error ^a	VIF	R ²	Power
A	0.2706	1.24813	0.1988	94.5 %
B	0.2912	1.03412	0.0330	91.1 %
AB	0.4576	1.25343	0.2022	55.9 %
A ²	0.4222	1.09754	0.0889	99.5 % ^b

Polar

Term	Standard Error ^a	VIF	R ²	Power
C	0.7675	6.39183	0.8436	24.2 %
D	0.6149	5.11388	0.8045	34.8 %
CD	1.07	5.97806	0.8327	14.7 %
C ²	0.7602	2.29919	0.5651	71.8 %

Correlation

Cartesian

	Intercept	A-Altitude (K ft)	B-Ground Range	AB	A ²
Intercept					
A-Altitude (K ft)					0.42
B-Ground Range					
AB					
A ²					

Polar

	Intercept	C-Slant Range	D-Depression Angle	CD	C ²
Intercept					
C-Slant Range					0.84
D-Depression Angle					
CD					
C ²					

Matrix Measures

Cartesian

Description	Value
Condition Number of Coefficient Matrix	2.83
Maximum Variance Mean	0.6819
Average Variance Mean	0.1314
Minimum Variance Mean	0.0517
G Efficiency	22.01
Scaled D-optimality Criterion	2.82
Determinant of (XX) ⁻¹	5.36077E-6
Trace of (XX) ⁻¹	0.6266
I (Cuboidal)	0.1315

Polar

Description	Value
Condition Number of Coefficient Matrix	30.83
Maximum Variance Mean	9.79
Average Variance Mean	0.7405
Minimum Variance Mean	0.0521
G Efficiency	1.60
Scaled D-optimality Criterion	5.23
Determinant of (XX) ⁻¹	1.16772E-4
Trace of (XX) ⁻¹	2.85
I (Cuboidal)	0.7471

Prediction Error

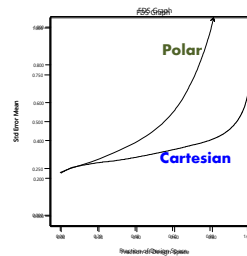
Model Mean
Standard Error
95% CI
95% PI

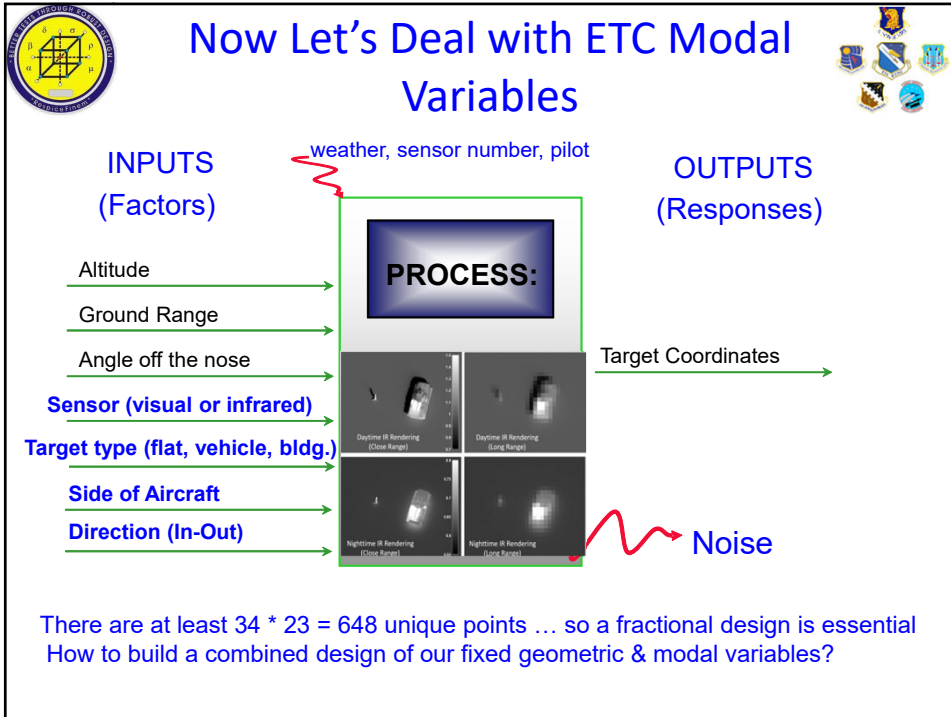
Model
Mean
Standard Error

95% CI
95% PI

Model
Mean
Standard Error

95% CI
95% PI





History of Experimental Design Gives Inspiration

- Blocks are (nearly) orthogonal sub-designs of modal variables!
- Random Balance Designs were first attempts at fractional general factorials (Now NOLH)
- Generate blocked designs in the modal variables
 - Basic geometric design is 32 runs
 - So blocks of 4 to 8 runs are conveniently sized
 - And geometric design is replicated 2-4 times usually
- Sequentially roll the nearly orthogonal modal blocks across the geometric design
- Latin Squares give us the patterns to roll
 - ABCD BCDA CDAB DABC ABCD...

	Machine 1	Machine 2	Machine 3	Machine 4
Operator 1	A	B	C	D
Operator 2	C	A	D	B
Operator 3	B	D	A	C
Operator 4	D	C	B	A

4x4 Latin Square

Random Balance Experimentation
F. E. Satterthwaite
Pages 111-137 | Published online: 30 Apr 2012



Academic review of some inspiring work by our progenitors...



- **Blocks*** (1925) preserve effect estimates by sacrificing some effects to measure/isolate nuisance variance
- **Latin Squares*** (1930's & earlier) are double blocking designs for (typically) a single effect
- **Random Balance+** designs (1956) fractionate multilevel categorical designs
- Montgomery^x (1988) introduces **Partially Aliased** designs by alternating the fractional block generators from one replicate to the next



Vol. 1, No. 2
Technometrics
May, 1959

Random Balance Experimentation

F. E. SATTERTHWAITE
National Engineering Service,
Falmouth, Mass.,
and Harvard College,
North Andover, Mass.

Random balance experimental designs for two-way factorial experiments of arbitrary order are described. The designs are shown to be orthogonal and balanced for the main effects and two-way interactions. The designs are shown to be orthogonal for the main effects and two-way interactions. The designs are shown to be orthogonal for the main effects and two-way interactions.

DESIGN AND ANALYSIS OF EXPERIMENTS

DOUGLAS C. MONTGOMERY

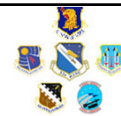


WILEY

*RA Fisher and the Design of Experiments, 1922-1926, JF Box, *The American Statistician* Vol. 34, No. 1 (Feb., 1980), pp. 1-7
 +Random Balance Experimentation Author(s): F. E. Satterthwaite, *Technometrics*, May, 1959, Vol. 1, No. 2, pp. 111-137
 X Ch 7.8 *Design and Analysis of Experiments*, DC Montgomery, 10th edition, Wiley 2020



Graphically – Alternate Modal Blocks Across Geometric Reps



Geometric Design				Modal Blks			
	Point	Altitude	Range				
Replicate I	1	10	15	inbound	Blk A		
	2	10	12		Blk B		
	3	10	8		Blk C		
	4	10	4		Blk D		
	5	10	4				
	6	10	8				
	7	10	12		outbound		
	32	15	15				
Replicate II	33	15	12	inbound	Blk B		
	34	15	8		Blk C		
	35	15	4		Blk D		
	36	15	4				
	37	15	8				
		64	15		15	outbound	Blk A
	Replicate III	65	15		12	inbound	Blk C
66		15	8	Blk D			
67		15	4	Blk A			
68		15	4				
69		15	8				
		96	10	15	outbound		Blk B

Modal Variables			
	Freq	Target	Side
Blk A	-1	1	1
Blk B	-1	1	-1
Blk C	1	-1	-1
Blk D	1	-1	1
Blk A	-1	-1	1
Blk B	1	1	1
Blk C	-1	-1	-1
Blk D	1	1	-1

- Result is a largely orthogonal crossed design



If Modal Variables Have Two Levels Then Blocks and Fractions are Simple



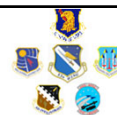
Table 3.15 Suggested Blocking Arrangements for the 2^k Factorial Design

Number of Factors, k	Number of Blocks, 2^p	Block Size, 2^{k-p}	Effects Chosen to Generate the Blocks	Interactions Confounded with Blocks
3	2	4	ABC	ABC
4	2	8	AB, AC	AB, AC, BC
	4	4	ABCD	ABCD
4	4	4	ABC, ACD	ABC, ACD, BD
	8	2	AB, BC, CD	AB, BC, CD, AC, BD, AD, ABCD
5	2	16	AB, AC, BC, CD, DE	AB, AC, BC, CD, DE, ABDE
	4	8	ABC, CDE	ABC, CDE, AC, ABCD, BD, ADE
5	8	4	ABE, BCE, CDE	ABE, BCE, CDE, AC, ABCD, BD, ADE
	16	2	AB, AC, CD, DE	All 2-factor and 4-factor interactions (15 effects)
6	2	32	ABCDEF	ABCDEF
	4	16	ABC, CDEF	ABC, CDEF, ABDE
6	8	8	ABEF, ABCD, ACE	ABEF, ABCD, ACE, BCF, BDE, CDEF, ADF
	16	4	ABF, ACF, BDF, DEF	ABE, ACF, BDF, DEF, BC, ABCD, ABDE, AD, ACDE, CE, BDF, BCDEF, ABCF, AEF, BE
7	2	64	AB, BC, CD, DE, EF	All 2-factor, 4-factor, and 6-factor interactions (31 effects)
	4	32	ABCDEF, G	ABCDEF, G
7	8	16	ABC, DEF, AFG	ABC, DEF, AFG, ABCDEF, DCFG, ADEG, BCDEFG
	16	8	ABD, EFG, CDE, ADG	ABCD, EFG, CDE, ADG, ABCDEFG, ABE, BCG, CDG, ADE, ACEG, ABFG, BCEF, BDEG, ACF, BDF
7	32	4	ABG, BCG, CDG, DEG, EFG	ABG, BCG, CDG, DEG, EFG, AC, BD, DE, DF, AE, BE, ABCD, ABDE, ABEF, BCDE, BCEF, CDEF, ABCDEFG, ADG, ACDEG, ACEFG, ABDFG, ABCEG, BEG, BDEFG, CFG, ADEF, ACDF, ABCE, AFG
	64	2	AB, BC, CD, DE, EF, FG	All 2-factor, 4-factor, and 6-factor interactions (63 effects)

- Remember this table? Fractional 2^k patterns date to 1940's
- Cochran & Cox Experimental Design 2nd Edition (1957)
- Design Expert Readily Generates 2^k Blocks with Generator Control



DX can generate alternate blocks of equal resolution



Replicates: 1 Blocks: 4 Center points per block: 0 Show Generators

Equal min aberration choices include:
AB ACD (BCD)+

BC ABD (ACD)
CD ABD (ABC) ...

Regular Two-Level Factorial Design

	2	3	4	5	6	7	8
4	2 ²	2 ³⁻¹					
8		2 ³	2 ⁴⁻¹ _{IV}	2 ⁵⁻²	2 ⁶⁻³	2 ⁷⁻⁴	
16			2 ⁴ _{IV}	2 ⁵⁻¹ _{IV}	2 ⁶⁻² _{IV}	2 ⁷⁻³ _{IV}	2 ⁸⁻⁴ _{IV}
32				2 ⁵ _V	2 ⁶⁻¹ _{IV}	2 ⁷⁻² _{IV}	2 ⁸⁻³ _{IV}

Factor Generators

Block Generators

1 = AB

2 = ACD

Save Factors

Set generators to defaults

Make generators editable

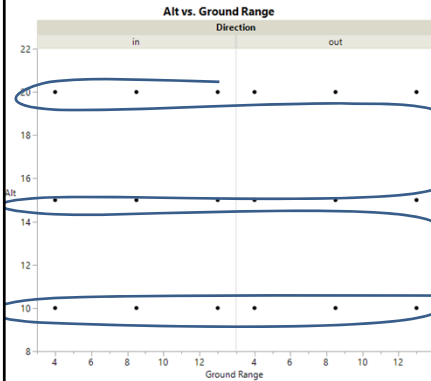
Use only the following factors when editing: A B C D

The default generators give a minimum aberration (the highest possible resolution) design. Editing the generators will change the aliasing patterns and may degrade the design's resolution -- edit with care!

- Exploits Montgomery's partial aliasing approach in text ch 7.8
- Take care with generalized interaction of block generators
- Avoid, for example, AB & ABC



Build two designs - replicated geometric factorial & 2^k modal factorial



Build 2^4 in 4 Blocks Res III With Modes and Targets

Block	Run	D-Side	E-Angle	F-Spectrum	G-Tgt
1	3	-1	1	1	1
1	13	1	-1	1	-1
1	6	1	1	-1	1
1	12	-1	-1	-1	-1
2	4	1	-1	1	1
2	16	-1	1	1	-1
2	7	-1	-1	-1	1
2	9	1	1	-1	-1
3	5	-1	1	-1	-1
3	10	1	-1	-1	1
3	2	1	1	1	-1
3	15	-1	-1	1	1
4	1	1	-1	-1	-1
4	11	-1	1	-1	1
4	8	-1	-1	1	-1
4	14	1	1	1	1

- Build sequential geometric design & replicates (96)
- Build complete modal variables in convenient block sizes
- Rotate or cross the blocks across the geo design



Cross-Block Design

- Left side 3 replicates of a full geometric design (N=32 each)
- Right columns are 2^4 modal variables in four convenient blocks
- Use Latin Square pattern to cross blocks with geometric replicates
- 1234 2341 3412 4123...
- Change block generators for next replicate

Point	Geometric Design (Polar)				Modal Design in Blocks				
	A-Depression	B-Slant Range	C-Direction	Direction	D-Side	E-Angle	F-Spectrum	G-Tgt	Block
1	20.9	15	1	in	-1	1	1	1	1
2	29.1	11	1	in	-1	1	-1	2	1
3	42.0	8	1	in	1	-1	-1	1	1
4	63.1	6	1	in	1	-1	1	2	1
5	89.7	5.3	-1	out	1	-1	1	1	2
6	63.1	6	-1	out	1	-1	-1	2	2
7	42.0	8	-1	out	-1	1	-1	1	2
8	29.1	11	-1	out	-1	1	1	2	2
9	14.3	15	1	in	1	1	1	2	3
10	19.7	11	1	in	1	1	-1	1	3
11	27.6	8	1	in	-1	-1	-1	2	3
12	67.8	4	1	in	-1	-1	1	1	3
13	89.6	3.7	-1	out	-1	-1	-1	1	4
14	47.8	5	-1	out	-1	-1	1	2	4
15	27.6	8	-1	out	1	1	-1	2	4
16	9.5	15	1	in	1	1	1	1	4
17	9.5	11	1	in	1	-1	1	1	2
18	9.5	8	1	in	1	-1	-1	2	2
19	14.3	5	1	in	-1	1	-1	1	2
20	38.1	2	1	in	-1	1	1	2	2
21	89.6	1.2	-1	out	1	1	1	2	3
22	38.1	2	-1	out	1	1	-1	1	3
23	14.3	5	-1	out	-1	-1	-1	2	3
24	10.2	7	-1	out	-1	-1	1	1	3
25	9.5	15	1	in	-1	-1	-1	1	4
26	13.0	11	1	in	-1	-1	1	2	4
27	20.7	7.0	1	in	1	1	-1	2	4
28	44.9	3.5	1	in	1	1	1	1	4
29	90.0	2.5	-1	out	-1	1	1	1	1
30	63.9	2.75	-1	out	-1	1	-1	2	1
31	29.6	5.0	-1	out	1	-1	-1	1	1
32	15.1	9.5	-1	out	1	-1	1	2	1
33	20.9	15	1	in	1	1	1	2	3
34	29.1	11	1	in	1	1	-1	1	3
35	42.0	8	1	in	-1	-1	-1	2	3
36	63.1	6	1	in	-1	-1	1	1	3
37	89.7	5.3	-1	out	-1	-1	-1	1	4
38	63.1	6	-1	out	-1	-1	1	2	4

If Modal Variables Are Multi-level Categoric or Odd Then Optimally Block

Block	Run	Factor 1 A/Target	Factor 2 B/Side	Factor 3 C/Frequency	Factor 4 D/Angles	Response 1 R1
Block 1	1	Box	Right	High	45	
Block 1	2	Truck	Right	Low	45	
Block 1	3	Truck	Left	Low	60	
Block 1	4	Road	Left	Low	45	
Block 1	5	Box	Left	High	10	
Block 1	6	Truck	Left	High	45	
Block 1	7	Road	Right	High	60	
Block 1	8	Truck	Left	High	10	
Block 2	9	Truck	Right	Low	60	
Block 2	10	Road	Right	High	45	
Block 2	11	Truck	Left	Low	45	
Block 2	12	Road	Left	Low	10	
Block 2	13	Box	Left	High	60	
Block 2	14	Box	Right	Low	10	
Block 2	15	Road	Left	Low	60	
Block 3	16	Box	Right	Low	60	
Block 3	17	Road	Left	High	60	
Block 3	18	Road	Right	Low	45	
Block 3	19	Road	Right	High	10	
Block 3	20	Box	Left	Low	10	
Block 3	21	Box	Left	High	45	
Block 3	22	Truck	Right	High	10	
Block 4	23	Road	Right	Low	10	
Block 4	24					
Block 4	25					
Block 4	26					
Block 4	27					
Block 4	28	Truck	Left	Low	10	
Block 4	29	Road	Left	High	45	
Block 5	30	Road	Right	Low	60	
Block 5	31	Box	Right	High	60	
Block 5	32	Road	Left	High	10	
Block 5	33	Box	Left	Low	45	
Block 5	34	Truck	Right	Low	10	
Block 5	35	Truck	Left	High	60	
Block 5	36	Truck	Right	High	45	

Optimal Design 1

Block	Run	Factor 1 A/Target	Factor 2 B/Side	Factor 3 C/Frequency	Factor 4 D/Angles	Response 1 R1
Block 1	1	Truck	Left	Low	60	
Block 1	2	Truck	Left	High	45	
Block 1	3	Box	Left	High	10	
Block 1	4	Box	Right	High	45	
Block 1	5	Truck	Right	Low	45	
Block 1	6	Road	Left	Low	45	
Block 1	7	Truck	Left	High	10	
Block 1	8	Road	Right	High	60	
Block 2	9	Box	Left	High	60	
Block 2	10	Road	Left	Low	10	
Block 2	11	Truck	Left	Low	45	
Block 2	12	Road	Right	High	45	
Block 2	13	Truck	Right	Low	60	
Block 2	14	Box	Right	Low	10	
Block 2	15	Road	Left	Low	60	
Block 3	16	Box	Left	Low	50	
Block 3	17	Box	Right	Low	60	
Block 3	18	Road	Right	Low	45	
Block 3	19	Truck	Right	High	10	
Block 3	20	Road	Left	High	60	
Block 3	21	Box	Left	High	45	
Block 3	22	Road	Right	High	10	
Block 4	23	Road	Left	High	45	
Block 4	24					
Block 4	25					
Block 4	26					
Block 4	27					
Block 4	28	Box	Right	Low	45	
Block 4	29	Truck	Left	Low	10	
Block 5	30	Box	Left	Low	45	
Block 5	31	Truck	Right	Low	10	
Block 5	32	Truck	Left	High	60	
Block 5	33	Road	Right	Low	60	
Block 5	34	Truck	Right	High	45	
Block 5	35	Box	Right	High	10	
Block 5	36	Box	Right	High	60	

Optimal Design 2

- 2² X 3² Design of Modal Variables N=36
- 5 Blocks of 7-8 Runs each
- In Geometric reps 2&3, use different sets 5 optimal blocks

96 Run Cross-Block Design Typical 2FI Evaluation

- Mild VIF inflation
- Mild 2FI and RSM covariances
- Flown these since circa 2018 – two dozen times
- Models confirm when flown on 4 ranges
- Models cross-validate nicely

Model Terms

Term	Standard Error*	VIF	R ²	Power
A	0.1332	1.13532	0.1192	99.9 %
B	0.1632	1.25105	0.2007	99.9 %
C	0.1794	1.30106	0.2314	99.9 %
D	0.1058	1.07118	0.0664	99.9 %
E[1]	0.1583			99.9 %
E[2]	0.1510			
F	0.1092	1.14409	0.1259	99.9 %
AB	0.1882	1.11911	0.1064	99.9 %
AC	0.2201	1.30829	0.2356	99.9 %
AD	0.1276	1.04193	0.0402	99.9 %
AE[1]	0.1953			99.9 %
AE[2]	0.1854			
AF	0.1301	1.08288	0.0765	99.9 %
BC	0.2804	1.428	0.2997	99.3 %
BD	0.1577	1.17885	0.1517	99.9 %
BE[1]	0.2429			99.9 %
BE[2]	0.2417			
BF	0.1630	1.24762	0.1985	99.9 %
CD	0.1716	1.19233	0.1613	99.9 %
CE[1]	0.2586			99.9 %
CE[2]	0.2648			
CF	0.1778	1.27779	0.2174	99.9 %
DE[1]	0.1562			99.9 %
DE[2]	0.1460			
DF	0.1070	1.09574	0.0874	99.9 %
E[1]F	0.1577			99.9 %
E[2]F	0.1555			

Pearson's r



Original Written Directions Circa January 2018

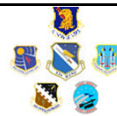


	GEOMETRY	MODES ETC.									
N=32 REP 1		<table border="0"> <tr><td>II</td><td>8</td><td>I 8</td></tr> <tr><td>III</td><td>8</td><td></td></tr> </table>	II	8	I 8	III	8				
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III	8	III 8									
I	8										
II	8										
<hr/>											
96											

THIS DESIGN REPEATS 1 BLOCK WITH EACH GEO REP.
 ASSIGN MUDAL POINTS TO EXTRA GEO POINTS RANDOMLY.
 4. DO REVERSE DESIGN EVAL



DX Enables Complex Custom Designs & Evaluations



The physical world of flight test offers endless challenges to our creativity... DX a great partner tool



Wanted: A Star Trek Transporter!

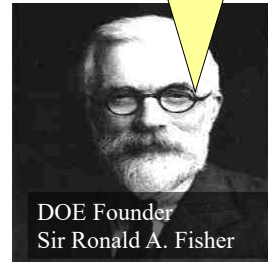


"To call in the statistician after the experiment is . . . asking him to perform a postmortem examination: he may be able to say what the experiment died of."
Address to Indian Statistical Congress, 1938.

US Space Force is working the tech

- In target location, we traverse the geometric space mated with blocked designs
- History of design development offered inspiration: Random Balance, Latin Squares, Partial Aliasing
- Algorithm easy to build & repeatable
- DX tools are critical – custom blocking, import & evaluate tailored designs
- Designs confirmed in experiments

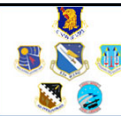
<https://www.bbc.com/news/world-us-canada-51245262>



DOE Founder
Sir Ronald A. Fisher



Wanted: A Transporter



Source: <https://www.vanityfair.com/hollywood/2017/12/star-trek-trouble-with-tribbles-50th-anniversary>